To minimize the time is to minimize the total distance the light travels going from A to B. The total distance is

$$D(x) = (x^2 + a^2)^{1/2} + ((c - x)^2 + b^2)^{1/2}.$$

Then $D'(x) = 0$ and $D(x)$ has it minimum when

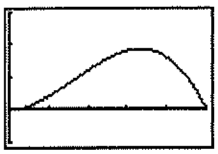
$x = \dfrac{ac}{a + b}$, or, $\dfrac{x}{a} = \dfrac{c}{a + b}$. It follows that

$c - x = \dfrac{bc}{a + b}$, or $\dfrac{c - x}{b} = \dfrac{c}{a + b}$. This means that the two triangles APR and BQR are similar, and the two angles must be equal.

59. **(a)** $\dfrac{dv}{dr} = cr(2r_0 - 3r)$ which is zero when

$r = \dfrac{2}{3}r_0$.

(b)



[0, 0.5] by [−0.01, 0.03]

61. $p(x) = 6x - (x^3 - 6x^2 + 15x)$, $x \geq 0$. This function has its maximum value at the points (0, 0) and (3, 0).

63. **(a)** $y'(0) = 0$ **(b)** $y'(-L) = 0$

(c) $y(0) = 0$, so $d = 0$. $y'(0) = 0$, so $c = 0$.

Then $y(-L) = -aL^3 + bL^2 = H$ and

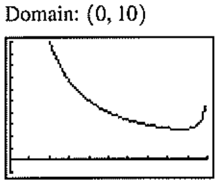
$y'(-L) = 3aL^2 - 2bL = 0$.

Solving, $a = 2\dfrac{H}{L^3}$ and $b = 3\dfrac{H}{L^2}$, which gives the equation shown.

65. **(a)** The x- and y-intercepts of the line through R and T are $x - \dfrac{a}{f'(x)}$ and $a - xf'(x)$ respectively.

The area of the triangle is the product of these two values.

(b) Domain: (0, 10)



[0, 10] by [−100, 1000]

The vertical asymptotes at $x = 0$ and $x = 10$ correspond to horizontal or vertical tangent lines, which do not form triangles.

(c) Height = 15, which is 3 times the y-coordinate of the center of the ellipse.

(d) Part (a) remains unchanged.

The domain is $(0, C)$ and the graph is similar.

The minimum area occurs when $x^2 = \dfrac{3C^2}{4}$. From this, it follows that the triangle has minimum area when its height is $3B$.

Section 5.5

Quick Review 5.5

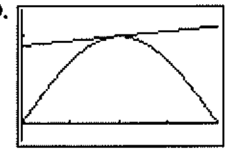
1. $2x \cos(x^2 + 1)$

3. $x \approx -0.567$

5. $y = x + 1$

7. **(a)** $x = -1$ **(b)** $x = -\dfrac{e + 1}{2e} \approx -0.684$

9.



$[0, \pi]$ by [−0.2, 1.3]

Exercises 5.5

1. **(a)** $L(x) = 10x - 13$

(b) Differs from the true value in absolute value by less than 10^{-1}

3. **(a)** $L(x) = 2$

(b) Differs from the true value in absolute value by less than 10^{-2}

5. **(a)** $L(x) = x - \pi$

(b) Differs from the true value in absolute value by less than 10^{-3}

7. $f(0) = 1$. Also, $f'(x) = k(1 + x)^{k-1}$, so $f'(0) = k$. This means the linearization at $x = 0$ is $L(x) = 1 + kx$.

9. **(a)** $1 - 6x$ **(b)** $2 + 2x$ **(c)** $1 - \dfrac{x}{2}$

11. $y = 10 + 0.05(x - 10)$, so $y = 10.05$

13. $y = 10 + (1/300)(x - 1000)$, so $y = 10 - 1/150 = 9.99\overline{3}$

15. **(a)** $dy = (3x^2 - 3)\,dx$

(b) $dy = 0.45$ at the given values

17. **(a)** $dy = (2x \ln x + x)\,dx$

(b) $dy = 0.01$ at the given values

19. **(a)** $dy = (\cos x)\,e^{\sin x}\,dx$

(b) $dy = 0.1$ at the given values

21. **(a)** $dy = \dfrac{dx}{(x + 1)^2}$

(b) $dy = 0.01$ at the given values

23. $-\dfrac{x}{\sqrt{1 - x^2}}\,dx$

25. $\dfrac{4}{1 + 16x^2}\,dx$

27. **(a)** 0.21 **(b)** 0.2 **(c)** 0.01

29. **(a)** $-\dfrac{2}{11}$ **(b)** $-\dfrac{1}{5}$ **(c)** $-\dfrac{1}{55}$

31. $\Delta V \approx 4\pi a^2 dr = 20\pi \text{ cm}^3$

33. $\Delta V \approx 3a^2\,dx = 15 \text{ cm}^3$

35. $\Delta V \approx 2\pi ah\,dr = \pi h \text{ cm}^3$

37. $2\pi(10)(0.1) \approx 6.3 \text{ in}^2$

39. $3(15)^2(0.2) \approx 135 \text{ cm}^3$

41. **(a)** $x + 1$ **(b)** $f(0.1) \approx 1.1$

(c) The actual value is less than 1.1, since the derivative is decreasing over the interval [0, 0.1].

43. The diameter grew $\dfrac{2}{\pi} \approx 0.6366$ in. The cross-section area grew about 10 in^2.

45. The side should be measured to within 1%.

47. $V = \pi r^2 h$ (where h is constant), so $\dfrac{dV}{V} = \dfrac{2\pi rh\,dr}{\pi r^2 h} = 2\dfrac{dr}{r} = 0.2\%$

49. Since $V = \dfrac{4}{3}\pi r^3$, we have $dV = 4\pi r^2 dr = 4\pi r^2\left(\dfrac{1}{16\pi}\right) = \dfrac{r^2}{4}$.

The volume error in each case is simply $\dfrac{r^2}{4} \text{ in}^3$.

Sphere Type	True Radius	Tape Error	Radius Error	Volume Error
Orange	2″	1/8″	$1/16\pi$″	1 in^3
Melon	4″	1/8″	$1/16\pi$″	4 in^3
Beach Ball	7″	1/8″	$1/16\pi$″	12.25 in^3

51. About 37.87 to 1

53. $x \approx 0.682328$

55. $x \approx 0.386237, 1.961569$

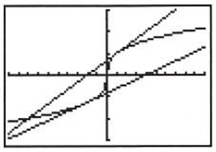
57. True. A look at the graph reveals the problem. The graph decreases after $x = 1$ toward a horizontal asymptote of $x = 0$, so the x-intercepts of the tangent lines keep getting bigger without approaching a zero.

59. B

61. D

63. If $f'(x_1) \neq 0$, then x_2 and all later approximations are equal to x_1.

65. $x_2 = -2$, $x_3 = 4$, $x_4 = -8$, and $x_5 = 16$;
$|x_n| = 2^{n-1}$.



[−10, 10] by [−3, 3]

67. Finding a zero of $\sin x$ by Newton's method would use the recursive formula $x_{n+1} = x_n - \dfrac{\sin(x_n)}{\cos(x_n)} = x_n - \tan x_n$, and that is exactly what the calculator would be doing. Any zero of $\sin x$ would be a multiple of π.

69.
$$\begin{aligned}\lim_{x\to 0}\frac{\tan x}{x} &= \lim_{x\to 0}\frac{\sin x/\cos x}{x}\\ &= \lim_{x\to 0}\left(\frac{1}{\cos x}\cdot\frac{\sin x}{x}\right)\\ &= \left(\lim_{x\to 0}\frac{1}{\cos x}\right)\left(\lim_{x\to 0}\frac{\sin x}{x}\right)\\ &= (1)(1) = 1.\end{aligned}$$

71. The linearization is $1 + \dfrac{3x}{2}$. It is the sum of the two individual linearizations.

Section 5.6

Quick Review 5.6

1. $\sqrt{74}$

3. $\dfrac{1-2y}{2x+2y-1}$

5. $2x\cos^2 y$

7. One possible answer: $x = -2 + 6t$, $y = 1 - 4t$, $0 \le t \le 1$.

9. One possible answer: $\pi/2 \le t \le 3\pi/2$

Exercises 5.6

1. $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt}$

3. **(a)** $\dfrac{dV}{dt} = \pi r^2\dfrac{dh}{dt}$
(b) $\dfrac{dV}{dt} = 2\pi rh\dfrac{dr}{dt}$
(c) $\dfrac{dV}{dt} = \pi r^2\dfrac{dh}{dt} + 2\pi rh\dfrac{dr}{dt}$

5. $\dfrac{ds}{dt} = \dfrac{x\dfrac{dx}{dt} + y\dfrac{dy}{dt} + z\dfrac{dz}{dt}}{\sqrt{x^2+y^2+z^2}}$

7. **(a)** 1 volt/sec **(b)** $-\dfrac{1}{3}$ amp/sec **(c)** $\dfrac{dV}{dt} = I\dfrac{dR}{dt} + R\dfrac{dI}{dt}$
(d) $\dfrac{dR}{dt} = \dfrac{3}{2}$ ohms/sec. R is increasing since $\dfrac{dR}{dt}$ is positive.

9. **(a)** $\dfrac{dA}{dt} = 14\text{ cm}^2/\text{sec}$ **(b)** $\dfrac{dP}{dt} = 0$ cm/sec **(c)** $\dfrac{dD}{dt} = -\dfrac{14}{13}$ cm/sec
(d) The area is increasing, because its derivative is positive. The perimeter is not changing, because its derivative is zero. The diagonal length is decreasing, because its derivative is negative.

11. **(a)** 1 ft/min **(b)** $40\pi\text{ ft}^2/\text{min}$

13. $\dfrac{dx}{dt} = \dfrac{3000}{\sqrt{51}}$ mph ≈ 420.084 mph

15. $\dfrac{19\pi}{2500} \approx 0.0239\text{ in}^3/\text{min}$

17. **(a)** $\dfrac{32}{9\pi} \approx 1.13$ cm/min **(b)** $-\dfrac{80}{3\pi} \approx -8.49$ cm/min

19. **(a)** 12 ft/sec **(b)** $-\dfrac{119}{2}\text{ ft}^2/\text{sec}$ **(c)** -1 radian/sec

21. **(a)** $\dfrac{5}{2}$ ft/sec **(b)** $-\dfrac{3}{20}$ radian/sec

23. **(a)** $\dfrac{24}{5}$ cm/sec **(b)** 0 cm/sec
(c) $-\dfrac{1200}{160.801} \approx -0.00746$ cm/sec

25. 1 radian/sec

27. $1.6\text{ cm}^2/\text{min}$

29. -3 ft/sec

31. In front: 2 radians/sec; Half second later: 1 radian/sec

33. 7.1 in./min **35.** 29.5 knots

37. **False.** Since $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt}$, the value of $\dfrac{dA}{dt}$ depends on r.

39. E **41.** B

43. **(a)** $\dfrac{dc}{dt} = 0.3$ $\dfrac{dr}{dt} = 0.9$ $\dfrac{dp}{dt} = 0.6$ $\dfrac{dc}{dt} = -1.5625$
(b) $\dfrac{dr}{dt} = 3.5$ $\dfrac{dp}{dt} = 5.0625$

45. **(a)** The point being plotted would correspond to a point on the edge of the wheel as the wheel turns.
(b) One possible answer:
$\theta = 16\pi t$, where t is in seconds.
(c) Assuming counterclockwise motion, the rates are as follows.
$\theta = \dfrac{\pi}{4}$; $\dfrac{dx}{dt} \approx -71.086$ ft/sec
$\dfrac{dy}{dt} \approx 71.086$ ft/sec
$\theta = \dfrac{\pi}{2}$; $\dfrac{dx}{dt} \approx -100.531$ ft/sec
$\dfrac{dy}{dt} = 0$ ft/sec
$\theta = \pi$: $\dfrac{dx}{dt} = 0$ ft/sec
$\dfrac{dy}{dt} \approx -100.531$ ft/sec

47. **(a)** 9% per year **(b)** Increasing at 1% per year

Quick Quiz (Sections 5.4–5.6)

1. B **3.** A

Review Exercises

1. Maximum: $\dfrac{4\sqrt{6}}{9}$ at $x = \dfrac{4}{3}$; minimum: -4 at $x = -2$

2. No global extrema

3. **(a)** $[-1, 0)$ and $[1, \infty)$
(b) $(-\infty, 1]$ and $(0, 1]$
(c) $(-\infty, 0)$ and $(0, \infty)$
(d) None
(e) Local minima at $(1, e)$ and $(-1, e)$
(f) None